Enrollment No: ____________________ **Exam Seat No:**____________________

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name : Discrete Mathematics

Subject Code : 4TE04DSM1 **Branch :** B.Tech(CE,IT)

Semester : 4 **Date :** 07/05/2016 **Time :** 2:30 To 5:30 **Marks :** 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

a) Define a complete digraph.
b) Give examples of isomorphic graphs.
c) Give an example of an edge simple path.
d) How many edges are there in a graph with 10 nodes each of degree 5?
e) Give an example of a non-commutative group.
f) Give an example of a cyclic group.
g) Define a normal subgroup of a group G.
h) Find the least and greatest element of the poset $\langle N, D\rangle$, if they exist.
i) Define: Bounded lattice.
j) Define: Sub algebra.
k) Prove that if $a=b$ then $ab'+a'b=0$.
l) In how many ways can symbols *a, b, c, d, e, e, e, e, e* be arranged so that no *e* is adiacent to another *e*?
m) State pigeonhole principle.
n) Define: Complement of a Fuzzy subset.

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) Let $\langle L,*,\oplus,',0,1\rangle$ be a complemented lattice and for any $a,b\in L$ then prove that $a\le b \Leftrightarrow a*b'=0 \Leftrightarrow b'\le a' \Leftrightarrow a'\oplus b=1$. **(07)**

b) i) By using mathematical induction prove that $1+3+5+...+(2n-1)=n^2$. **(07)**

ii) Solve the recurrence relation $a_{n+1}-2a_n=5;\ n\ge 0;\ a_0=1$.

Q-3 Attempt all questions **(14)**

a) Prove that $\langle P(\{a,b,c\}),\subseteq\rangle$ is a complemented lattice and also draw the Hasse diagram of it. **(07)**

b) Let $\langle L,\leq\rangle$ be a lattice and $a,b,c\in L$ then show that the following are equivalent. **(07)**

i) $a*(b\oplus c)=(a*b)\oplus(a*c)$

ii) $a\oplus(b*c)=(a\oplus b)*(a\oplus c)$

Q-4 Attempt all questions **(14)**

a) For a lattice $\langle S_{90},D\rangle$, answer the following questions: **(07)**

i) Find cover of each element and draw the Hasse diagram.

ii) Find lower bound, upper bound, greatest lower bound, least upper bound of $A=\{3,5,6\}$.

iii) Find the least and greatest element of it.

b) Let $E=\{a,b,c,d,e\}$, $\underset{\sim}{A}=\{(a,0.3),(b,0.8),(c,0.5),(d,0.1),(e,0.9)\}$, $\underset{\sim}{B}=\{(a,0.7),(b,0.6),(c,0.4),(d,0.2),(e,0.1)\}$ then find the following: **(07)**

1) $\underset{\sim}{A}\cup\underset{\sim}{B}$ 2) $\underset{\sim}{A}\cdot\underset{\sim}{B}$ 3) $\underset{\sim}{A}\hat{+}\underset{\sim}{B}$ 4) $\underset{\sim}{A}-\underset{\sim}{B}$ 5) $\underset{\sim}{A}\cap\underset{\sim}{B}$ 6) $\left(\underset{\sim}{A}'\right)'$ 7) $\underset{\sim}{B}'$

Q-5 Attempt all questions **(14)**

a) State and prove Stone's representation theorem. **(10)**

b) Obtain the sum of product canonical form of the Boolean expression in three variables **(04)**

$\alpha(x,y,z)=y'+\left[z'+x+(yz)'\right](z+x'y)$.

Q-6 Attempt all questions **(14)**

a) Show that the sum of indegrees of all the nodes of a simple digraph is equal to the sum of outdegrees of all its nodes, and that this sum is equal to the number of edges of the graph. Explain this statement by a simple example. **(05)**

b) For a simple digraph G = <V, E>, where V = {1, 2, 3, 4} and E = {<1, 4>, <1, 2>, <2, 4>, <2, 3>, <3, 4>, <4, 3>}, give matrix representation A of G with ordering 1, 2, 3, 4. Also draw the directional dual of G and show that its matrix representation is A^T. **(05)**

c) List the ways in which a directed tree can be represented graphically. Express the algebraic expression ab + (c + d/e)·f in a directed tree. **(04)**

Q-7 Attempt all questions **(14)**

a) Define: (1) directed tree; and (2) descendent of a node and son of a node with examples. **(05)**

b) Define a subgroup of a group G. Let H be a nonempty subset of a group G. Show that H is a subgroup of G if and only if $ab^{-1}\in H$, whenever a, b $\in$ H. **(05)**

c) Symbolize the statement "given any positive integer, there is a greater positive integer" with and without universe of discourse. **(04)**

Q-8 Attempt all questions **(14)**

a) Show that the set Q \ {-1} is an abelian group with respect to the binary operation $a*b=a+b+ab,$ for all a, b ∈ G. **(05)**

b) State and prove the Lagrange's theorem. **(05)**

c) Let G = Q^+, $a*b=\frac{ab}{2}$, Find the identity element and a^{-1} in G. **(04)**